Abstract:

Equity markets are one of the most complicated systems, and modeling them in terms of dynamical equations is nearly difficult. The fundamental reason for this is that stock prices are influenced by a number of unknown factors such as economic conditions, corporate policy changes, supply and demand among investors, and so on. These variables are continually changing, making stock markets extremely volatile. Stock price prediction is a classic nonstationary pattern recognition issue in Machine Learning. There has been a lot of study through using Artificial Intelligence and Machine Learning techniques like Artificial Neural Networks, Fuzzy logic, and Support Vector Regression to forecast the behavior of stocks based on their previous performance. Hidden Markov Models are a way for evaluating the stock markets that isn't as well-known as the others. As a result, in this term paper, I will focus on Hidden Markov Models in order to discover the regime as well as anticipate stock behavior. Also, an investor's performance, particularly in the stock market, is dependent in big part on the decisions taken, which in turn is dependent in major part on how well informed one is in stock research. The Markov chain model was used to assess and predict the three states of stock price change: growth, drop, and remain unchanged.

1. Introduction

For recognizing patterns in random processes, Hidden Markov Models offer a strong probabilistic framework . The essential principle behind this model is that the likelihood of observations is determined by system states that are 'hidden' from the observer. The transition from one state to the next is a Markov Process, which means that the next state is only dependent on the current one, hence the name Hidden Markov Models. In HMMs, states are always discrete, but observations might be discrete, continuous, or both. Stock markets can be viewed as a Hidden Markov Process where the investor can only observe the stock prices and the underlying states which are driving the stock prices are unknown.

Investors face a constant problem in determining the financial markets' frequent behavior as a result of changes in government policies, negative news, regulatory environments, and other macroeconomic influences. Market Regime is a term used to describe such eras. Changes in asset returns' means, variances, autocorrelation, and covariances result from these distinct regimes. This has an impact on the effectiveness of stationarity-based time series approaches. In this term paper, I will use Hidden Market Models to find market regimes, and then I will use the same model to assess its efficacy in predicting stock prices. To develop a prototype, I am considering the following observations: daily open, close, high, and low for three stocks: Apple Inc., Comcast In. In this project, I'll use a multivariate Gaussian distribution to represent our observations.

Then, even when the market is in a random walk situation, I'll look at the transition matrix of these companies' share returns and try to figure out the transition matrix and equilibrium matrix. For this, I'll use the "Adj Close" observations for the stocks of these three companies.

2. Theoretical Framework

2.1 Hidden Markov Model: HMM is a generative probabilistic model in which the system is considered to be transitioning in certain number of states. The state transition is a Markov Process and hence can be defined by a matrix of state transition probabilities. As previously mentioned, the state sequence is not directly visible but some of the state dynamics can be observed. The corresponding joint density function for the HMM is given by (again using notation from Murphy (2012):

P(X1:T) = p((X1)p(X2|X1)p(X3 | X2)…

=p(X1)

In the first line this states that the joint probability of seeing the full set of hidden states and observations is equal to the probability of simply seeing the hidden states multiplied by the probability of seeing the observations, conditional on the states. This makes sense as the observations cannot affect the states, but the hidden states do indirectly affect the observations.

The second line splits these two distributions into transition functions. The transition function for the states is given by p(zt∣zt−1) while that for the observations (which depend upon the states) is given by p(xt∣zt).

2.2 Hidden Markov Models for Financial Time Series analysis :

Hidden Markov Models have been a powerful tool for analyzing non-stationary systems. Stock Markets are non-stationary systems and the observations are continuous in nature. Consider 𝑂𝑡 be a vector of four elements- daily close, open, high and low and 𝑆𝑡 to be the state on day 𝑡. The state 𝑆𝑡 can be one of the assumed states. The figure 1 shows a typical Hidden Markov Process.

Figure 1: Hidden Markov Process

Since the vector 𝑂𝑡 takes real values, observations can be modelled as Multivariate Gaussian distributed. The observations are assumed to independent whereas the elements of an observation may be correlated. The state 𝑆𝑡 can take only discrete values since HMM is a finite state machine. Let us now define some terminologies that are used to define Hidden Markov Models. We will be using the same notations throughout this project.

= Observation on day , daily close, daily open, daily high, daily low

= State on the day

= Number of observations

= Latency

=Number of States ()

=Observation sequence

=Initial State Probability

where is the state transition probability from state to

=mean of the multivariate Gaussian distribution of observation of state

=Covariance matrix of the distribution of observation of state

The Hidden Markov Model can be represented as

In this paper, I will use this model for the regime detection of stock market and to analyze the price behavior of the stated companies’ stock.

Markov Chain Model: A Markov chain is a set of trials with a finite number of states and probabilities that are known. P, where P represents the probability of advancing from state I to state j, or simply expressed, a stochastic process that is based on the immediate outcome rather than on the past. It can be thought of as a series of transitions between different states, with the probabilities associated with each transition based solely on the current state and not on how the process got there, and the probabilities associated with the transitions between the states remaining constant over time. When the current outcome is known, knowledge from previous trials has little bearing on the likelihood of future events. The Markov chain model can then be said to be a sequence of consecutive trials such that,

P{Xn = j/Xn-1 = in-1,…, X0 = i0} = P{Xn= j/ X n-1 = in-1}

P{xn= j} = Pj (n) is the absolute probability of outcome Pj, , j = 1,2,3,… is a system of events (actually set of outcomes at any trial) that are mutually exclusive (Voskoglou, 1994; Hoppensteadt, 1992).An important class of Markov chain model is that of which the transition probabilities are independent of n, we have P{xn = j/xn-1= i}= Pij which is a homogenous Markov chain where the order of the subscripts in Pij corresponds to the direction of the transition i.e i j. Hence we have ∑ Pij = 1 and Pij ≥ 0, Since for any fixed i, the transition probability Pij will form a probability distribution. If the limiting distribution of xn as n ∞ exist, the transition probabilities are most conveniently handled in matrix form as P = Pij i.e

P11 P12 . . . P1n

P21 P22 .

. . . P2n

P =

. . . . .

. . . . .

. . . . .

Pn1 Pn2 . . . Pnn

And this is referred to as the transition matrix, which depends on the number of states involved and may be finite or infinite (Hamilton, 1989; Michael, 2005).

The absolute probabilities at any stage where n is greater than unity is determined by the used of n-step transition probabilities i.e. In matrix terms, let p be the transition matrix of the Markov chain, then

P 1= PP(0) (for n=1)

Also P2= PP1= P(PP(0) ) = P2P (0) (for n =2)

And in general P (n) = PnP (0)

With this formulation, I will model and calculate the transition matrix and the equilibrium matrix in this paper.

1. Literature Review:

In many financial problems, the states of a system can be modeled as a Markov chain in which each state depends on the previous state in a non-deterministic way. In a hidden Markov model (HMM), these states are invisible, while observations (the inputs of the model), which depend on the states, are visible. An observation at time *t* of an HMM has a certain probability distribution corresponding to a possible state.

Researchers have applied HMM for analyzing economic trends. HMM was used in with three states to predict currency crises in six developing countries. Hassan and Nath used HMM to forecast the stock price for interrelated markets. Kritzman, Page and Turkington applied HMM with two states to predict regimes in market turbulence, inflation and the industrial production index. Guidolin, and Timmermann used HMM with four states and multiple observations to study asset allocation decisions based on regime switching in asset returns. Ang and Bekaert applied a regime shift model (an other name for HMM) for international asset allocation. Nguyen used HMM with both single and multiple observations to forecast economic regimes and stock prices. Nobakht, Joseph and Loni implemented HMM using multiple observation data (open, close, low, high) prices of a stock to predict its close prices. However, the momenta of a stock depends on many different factors, such as the corporate financial condition and management and the overall economy and industry conditions. These factors and corresponding stock returns vary widely over different macro regimes. In addition, long-term stock investments’ returns depend on the trends of these economic factors. Therefore, in this paper, I have tried to analyze approach of HMM through which the investors can find the volatility state and also applying this model, I have tried to predict the stock behavior of three companies.

However, The stock market forecasting is marked more by its failure than by its successes since stock prices reflect the judgments and expectations of investors, based on the information available. If things look good, the price moves upward so quickly that recipient of cheerful information has little or no time to act upon it. Remarkably, efforts have been made to apply econometric techniques of model building in the prediction of stock prices in an effort to demonstrate that the market fluctuations are essentially unpredictable (Bernstein and Bostain, 1974; Black, 1971; Brealey and Myers, 1996; Buhlmann, 2005). Fama and French (1988) have argued that there are long term pattern in stock prices with several years of upswing followed by more sluggish periods. According to Fama (1965; 1995), a stock market where successive, price changes in individual securities are independent is, by their definition a random walk market. According to Kendal (1953), stock prices following a random walk implies that the price changes are as independent of one another as the gains and losses. In this study, the random walk approach is presented with the specific aim of giving a definite description of the Nigerian stock market prices. In a world without interest rates, idealized stock prices should be martingales. This is one way of formulating the so called efficient market hypothesis (Buhlmann, 2005; Fama, 1965; 1995)

1. Model and Results

Hidden Markov Model for Regime Detection:

|  |  |  |  |
| --- | --- | --- | --- |
| Specifications | 0th hidden state | 1th hidden state | 2th hidden state |
| Mean | 7.91014566e-04 | 0.00148548 | 9.05984448e-04 |
| Variance | 3.23204848e-04 | 3.08223754e-04 | 0.00138042 |

We know that time series exhibit temporary periods where the expected means and variances are stable through time. These periods or regimes can be likened to hidden states. If that's the case, then all I need are observable variables whose behavior allows us to infer the true hidden state(s). If i can better estimate an asset's most likely regime, including the associated means and variances, then the predictive models become more adaptable and will likely improve. I will use the **sklearn's GaussianMixture** to fit a model that estimates these regimes. The important takeaway is that mixture models implement a closely related unsupervised form of density estimation. It makes use of the expectation-maximization algorithm to estimate the means and covariances of the hidden states (regimes). I have arbitrarily classify the regimes as High, Neutral and Low Volatility and set the number of components to three.

Table: Mean and Variances of each hidden states of Google Inc

In the above image, I've highlighted each regime's daily expected mean and variance of Google Inc. returns. It appears the 1th hidden state is our low volatility regime. Note that the 1th hidden state has the largest expected return and the smallest variance. The 0th hidden state is the neutral volatility regime with the second largest return and variance. Lastly the 2th hidden state is high volatility regime. We can see the expected return is positive but the variance is the largest of the group.

|  |  |  |  |
| --- | --- | --- | --- |
| Specifications | 0th hidden state | 1th hidden state | 2th hidden state |
| Mean | 0.00063252 | 9.67519722e-04 | 2.51123522e-04 |
| Variance | 2.81538338e-04 | 1.58789458e-04 | 0.00127313 |

To include about the Apple Inc and Comcast Corp., again the highest volatility we can say is 2th hidden state:

|  |  |  |  |
| --- | --- | --- | --- |
| Specifications | 0th hidden state | 1th hidden state | 2th hidden state |
| Mean | 6.20135224e-04 | 5.43647511e-04 | -1.33789919e-03 |
| Variance | 2.83616732e-04 | 1.46794423e-04 | 0.00133433 |

A picture containing chart

Description automatically generatedDiagram

Description automatically generatedTo visualize the volatility and the above scenarios, I have plotted a graph:

Figure: States of Comcast Corp. Figure: States of Apple Inc.

Diagram

Description automatically generated

Figure: States of Google Inc.

Chart

Description automatically generatedChart, line chart

Description automatically generatedChart

Description automatically generatedAs we have defined each of the three defined states for the three organizations, and also we have seen that there are huge volatilities in the states, now we will see the regimes that in particular which year a company is having these three different states.

Figure: Regime Detection of Comcast Corp., Apple Inc., Google Inc.

From the above graph, we can see that, basically in 2008 and in 2020 three of these companies had faced the most volatilities which is actually clear as in 2008, there was a global financial crisis happened and in 2020-2021, due to the COVID-19, the share market again faced extreme volatilities.

Prediction of Stock Prices:

The main idea for predicting the next day’s stock price is to calculate the log-likelihood of 𝐾 previous observations and comparing it with the log-likelihood of all the previous sub-sequences of same size by shifting the window by one day in the direction of past data. After that, I have identified a day in the past whose loglikelihood of its 𝐾 previous observations is the closest to the sub-sequence whose next day’s price is to be predicted.

where

I then calculated the differential price change from the identified day to its next day. This change is then added to the current day’s price to get our next day’s prediction. Subsequently, after getting the true observation, I have included it to our dataset and retune our model parameters in order to ensure that our model doesn’t diverge. In short, I fix the size of our sub-sequence and locate another sub-sequence from the past data which exhibits a similar pattern. After all these, I have made the map the behavior of the identified sub-sequence to the subsequence being used for prediction. In order to select a model with optimal number of states, I train a set of models by varying the number of states (𝑁). Next, I calculate the negative loglikelihood of the training data used for each of the models and chose the model which has the lowest value. However, this approach tends to prefer a complex model implying that the number of states chosen may tend to a higher value and might result in overfitting. In order avoid this problem, I have added a penalty term to the negative log likelihood. Depending on the penalty term chosen, I imposeD restrictions on the model at varying degree.

The performance metric that I used in this project is Mean Absolute Percentage Error (MAPE) which is defined as:

MAPE (Mean Absolute Percentage Error)

In this project, the main objective was to determine the efficiency of HMMs in predicting the stock prices. I have used hmmlearn, an open source python library to train the model and calculate the likelihood of the observations. The stocks that we selected are Google Inc., Apple Inc., Comcast Corp. I have used opening price, closing price, high and low as features for the past working days of 10 years when the market was open. I kept aside the recent 100 observations for testing and used rest of the observations for training the model. Next, I predicted the prices for the past 100 days, starting from the 100th day and then using its true observation to retune the model for predicting the 99th day and so on. Therefore, every time I retune the model, the number of training samples will increase by one. First, I implemented using fixed model i.e. by fixing the number of states to four. Figures shows the stock price predictions for Google using  
HMM with four states. I have calculated the MAPE and plotted the predictions and the actual prices to compare the results.

Implementation using 4 state model for Apple Inc for next 100 days.

Chart, line chart

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Figure: Prediction of the 100 days of Apple Inc.

Chart, line chart

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I have calculated the MAPE and plotted the predictions on the same plots of HMM. Table shows the MAPE values for all the three stocks:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Company | **Close** | **Open** | **High** | **Low** |
| Google Inc. | 0.0057 | 0.0057 | 0.0061 | 0.0058 |
| Apple Inc. | 0.00888693 | 0.00980546 | 0.00920139 | 0.00866747 |
| Comcast Corp. | 0.00781854 | 0.00942235 | 0.0075632 | 0.00799607 |

From the above table, it can be said that HMM captures the volatility of the stock prices as well as HMM can work better for the stocks with high volatility.

**Finding Equilibrium Matrix:**

**Share Price Movement:**

This study focused on the top three corporations in the US stock market. Data on the share prices of the mentioned three companies was gathered from Yahoo Finance's daily list from 2002 to 2022 (except for the random walk equilibrium matrix). The transition from one state to another (that is, the share price movement pattern, which could be that a decrease in price is followed by another decrease, a decrease is followed by unchanged, a decrease is followed by an increase, and so on) was observed from the data collected, and the results for each company under study were compiled using Python (Jupyter Notebook) as follows (below represents only Apple Inc.;; Google Inc. and Comcast Corp. will be found in the Appendix.).

| Date | **High** | **Low** | **Open** | **Close** | **Volume** | **Adj Close** | **state** | **Prior state** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **2022-04-25** | 163.169998 | 158.460007 | 161.119995 | 162.880005 | 96046400.0 | 162.880005 | Upside | Downside |
| **2022-04-26** | 162.339996 | 156.720001 | 162.250000 | 156.800003 | 95623200.0 | 156.800003 | Downside | Upside |
| **2022-04-27** | 159.789993 | 155.380005 | 155.910004 | 156.570007 | 88063200.0 | 156.570007 | Downside | Downside |
| **2022-04-28** | 164.520004 | 158.929993 | 159.250000 | 163.639999 | 130216800.0 | 163.639999 | Upside | Downside |
| **2022-04-29** | 166.199997 | 157.250000 | 161.839996 | 157.649994 | 124911916.0 | 157.649994 | Downside | Upside |

Table: **The Share Price Movement of Apple Inc.**

## Construction of the Markov Chain Using Python

## Apple Inc., Google Inc., and Comcast Corp Inc. can all have three states using the Markov Chain:

## Upside: Today's pricing is higher than yesterday's price.

## Downside:  today's pricing is lower than yesterday's price.

## Consolidation: The price is the same as the day before.

The first step in obtaining the states in our data frame is to calculate the daily return. Then, based on the return, I will utilize a function to identify the possible states. Now, defining Consolidation as a condition in which there is literally no movement on a given day is realistically untenable. As a result, I have kept the legroom to a bare minimum. Even if the movement is limited to a short range, it is still referred to be a consolidation condition.

|  |  |  |
| --- | --- | --- |
| State  Prior State | Downside | Upside |
| Consolidation | 0.000000 | 1.000000 |
| Downside | 0.495127 | 0.504873 |
| Upside | 0.508036 | 0.491964 |

However, in a volatility market like stock market, there is no current state of Consolidation. As a result, instead of a 3x3 matrix, it produces a 3x2 matrix. When all of the values in a column in a matrix calculation are 0, the column can be omitted. Thus, moving forward I will use only two states which are downside and upside.

Like, we know that, Transition Matrix shows the probability of the occurrence instead of the number of occurrences. That’s why it is also called “Initial Probability Matrix”. Let’s consider time as t. Basically, “Transition Matrix” is the probability matrix at t=0. Thus, if I build the Markov Chain by multiplying this transition matrix by itself to obtain the probability matrix in t=1 which would allow me to make one-day forecasts. With the formula of  π\_0 = t\_0 = A, I am going to calculate the Markov Chain similarly. Below represents the Markov chain of three different companies at t = 0:

Transition Matrix for Google Inc:

|  |  |  |
| --- | --- | --- |
| State  Prior State | Downside | Upside |
| Downside | 0.470894 | 0.529106 |
| Upside | 0.477147 | 0.522853 |

Transition Matrix for Apple Inc.:

|  |  |  |
| --- | --- | --- |
| State  Prior State | Downside | Upside |
| Downside | 0.469959 | 0.530041 |
| Upside | 0.479345 | 0.520655 |

Transition Matrix for Comcast Inc..:

|  |  |  |
| --- | --- | --- |
| State  Prior State | Downside | Upside |
| Downside | 0.479636 | 0.520364 |
| Upside | 0.508501 | 0.491499 |

This is the Initial Probability Matrix. With, t =1 I will be able to get the one day forecasts, and with the same process I will be able to see the foreseeable future point from which the state will not change and will remain equilibrium. In the next part, I will cover to find the equilibrium point with random walk for these three companies.

**Finding Equilibrium Matrix with Random Walk: using Python**

To find out the equilibrium matrix we can iterate the process up to the probabilities don’t change more. I have tried to do a random walk using out Yahoo Finance Library and use 10000 days. As We theoretically can not have ∞ days of data, I have taken 10,000 days from the start date to the end date. Now, as we are supposedly “walking” randomly from one day to another, I consider our 10,000 days of walking as ∞ steps.  To find the equilibrium point, I have taken the previous 10000 days in order to find the equilibrium point of three companies.

|  |  |  |  |
| --- | --- | --- | --- |
| Specification | Google Inc. | Apple Inc. | Comcast Inc. |
| Equilibrium Matrix Number: | 11 | 9 | 12 |

From the table, the result represents that after 11 days, we will get our equilibrium point for Google Inc and after 9 and 12 days, we will get our equilibrium point for Apple Inc and Comcast Inc respectively. However, the value of the matrix matters much which we will be found next.

Equilibrium Matrix of Google:

|  |  |  |
| --- | --- | --- |
| State  Prior State | Downside | Upside |
| Downside | 0.461052 | 0.538948 |
| Upside | 0.461052 | 0.538948 |

The result meant that, for Google Inc., if we had a Downside day today, tomorrow there is 46.1052% of probability of having downside day and again if we had a downside day today, tomorrow there is a chance of having upside day is of 53.8948%.

Equilibrium Matrix of Apple:

|  |  |  |
| --- | --- | --- |
| State  Prior State | Downside | Upside |
| Downside | 0.487522 | 0.512478 |
| Upside | 0.487522 | 0.512478 |

Similarly, for Apple Inc., if we had a Downside day today, tomorrow there is 48.7522% of probability of having downside day and again if we had a downside day today, tomorrow there is a chance of having upside day is of 51.2478%.

Equilibrium Matrix of Comcast Inc.:

|  |  |  |
| --- | --- | --- |
| State  Prior State | Downside | Upside |
| Downside | 0.501523 | 0.498477 |
| Upside | 0.498477 | 0.498477 |

The result meant that, for Comcast Inc., if we had a Downside day today, tomorrow there is 50.152% of probability of having downside day and again if we had a downside day today, tomorrow there is a chance of having upside day is of 49.8477%.

The result from this analysis provided evidence to show that individual stocks generally satisfy the random walk hypothesis by using historical stock price data. The study also demonstrated that the US stock market is extreme volatile.

Conclusion:

This study describes the theory of random walks and some of the important issues it raises concerning the stock market. A market where there are successive price changes in individual securities is simply a random walk market. The study shows that the stock price changes have no memory of the past history and that no investor can alter the fairness or unfairness of a stock price as defined by expectation. We can formalize fairness or unfairness in terms of a random walk {Sn, nEIN}. The study shows that stock prices is but a martingale and all that investors can do is to narrow discrepancies between fairness and unfairness in a way that high probabilities of small gains may be exchanged for low probabilities of large gains. The investor is not allowed at any time to invest in the market in a way which might involve his losing more than his investment. There expected fortune after trial n + 1 is the same as their present fortune as long as the particular investment remains fair or unfair.